

The last intercalation

Some remarks on the Persian calendars

The main source mentioning the last intercalation in the Sasanian times is Bērōnī's accounts in his early work on the chronology of ancient nations, the *Vestiges of the Past Centuries*, and also in his later masterpiece consisting of an encyclopedia of astronomical sciences, the *Canon Masudicus*. However under the pretext that there is a contradiction in Bērōnī's accounts as he has given two different "dates" for the same alleged intercalation, i.e. in the reign of Yazdegird I and in that of Pērōz, the historical existence of the intercalary cycle in the Persian calendric system has been thoroughly discredited. It is surprising to see the non-attentive reading of Bērōnī's accounts of the intercalation of the Persian calendar in its history; and, moreover, if we review most of the modern authors on the subject, their lack of familiarity with both the Perso-Aryan vision and the elements of Persian calendars is astounding.

In his *Chronology*, Bērōnī draws forth his account from the Mazdayasnian books which treat of the calendar of Persian festivals¹, and also from the set of astronomical tables of the Persian astronomers, the *Zīg ī Šahriyārān*²; his account is not a mere translation of Persian texts, but it is often accompanied by a commentary that could be used as a basis for a mathematical assessment. In his *Canon*, Bērōnī summarizes his previous account while correcting some minor errors.

The present article deals with the question of the last intercalation made by Persian astronomers as described by Bērōnī.

¹ . See Ch. IX.1 :

« و قد وجدنا كتاب زادويه بن شاهويه و كتاب خرشيد بن زيار موبد اصفهان و كتاب محمد بن بھرام بن مطيار، تشتمل من هذه المعاني علي شيء فيه غنية، و ليس فيه اختلاف اختلال؛ فنقلنا نحن ما فيها الي ما اثبتناه هاهنا، و ركبنا بعضها ببعض، و جمعنا أكثر ما قيل فيها.» الآثار الباقية عن القرون الخالية، ابوربحان البيروني، پ. ادكائي، تهران، ۱۳۸۰، ۲۶۳.

² . For example, in the first chapter on the nature of day and night (I.3), he refers to the author of the *Zīg ī Šahriyārān Šāh* as for the beginning of the day at midnight.

« وبعضهم آثر التّصف الخفّ من فلك نصف النهار، فابتدأ بهما من نصف الليل، كصاحب زيح شهرياران الشاه

... « الآثار، ۸.

§ 1. Perso-Aryan calendars are anchored in a double helix structure which reflects the connection between Daēnā (Pers. *dēn*) ‘religion, vision’ and Xšaθra (Pers. *šahr*) ‘royal power, kingdom’.

§ 1.1. The calendar pertaining to Daēnā represents Time as a harmonious course, and the time unit as a constant year consisting of 12 months of 30 days each and 5 additional days; then the starting point of this year form of always 365 days length moves in succession, through all the seasons.³ The first month is called Daθuš (Cappadocian *Δαθουσα*, Persic *Day*), that is, the creator, Ahura Mazdā –in later times, it was considered as the 10th month of the year. The five additional days (Pers. *Panzag*, *Andargāh* ≈ *ἐπαγομένων*) are placed at the end of the 9th month, Spəntā Ārmaiti (Capp. *Σονδαρα*, Pers. *Spendārmed*).⁴

$$1 \text{ y} = 365^{\text{d}}$$

The year with no intercalation serves religious purposes, and the calendar of the festivals is related to it; it is referred to, by the Persian astronomers, simply as the “calulatory” or “calculated” year (*ušmurdīg*)⁵. The later authors on the Persian calendric system take the religious year of the Persians without intercalation as the “civil” year, and the civil year with intercalary month(s) as the “religious” year. One source of confusion is this that they assume that the Persian 365-day calendar must have been borrowed from the Egyptian calendar, and the Egyptian fixed 365-day year was called, for example by the Roman author Censorinus (3rd Century) in his *De Die Natali* (Chapter 18: on various calendars of the ancient world), the

³ . Before the coming of the Daēnā Māzdayasni the length of the year was 360 days. The Persians attributed this year form to the hero-kings who bore the title *paraδāta* (Pers. *pēšdād*). We find the same form in use by another branch of the Aryans, the Gulish (called by Muslims the *Kāfir*). See my « Imrā, le dieu des Gulois », *Er*, 2002.

⁴ . Cf. VD 184 andar panzag dā vahištōišť kū hamaspasmēdēm ī māh spendarmed andar yazišn ud niyāyišn ud drōn sāxtan nē šāyed xʷandan. cē ēn panz rōz az māh judāg dāšt ēsted. cōn az abestāg ī nīkādum pēdāg:
*āaṭ aētahe pañca ayara hamaspaθmaēdaēm paiti ratūm spəntayā
ārmatoišť māñhō nōiṭ frasarvayōiṭ.*

⁵ . Cf. Dk iii, M 402 ud rōz-vihēzagīg ī xʷad ast hān ī ušmurdīg sāl ōz ud kār abar nōgrōz ud mihragān ud abārīg jašnīhā kahvan ast ī az bundahišn.

“civil” year.⁶ However, É. Dulaurier, in his essay on the Armenian chronology, understood the sacred aspect of the *annus vagus*.⁷

§ 1.2. The religious calendric year was in continuous use, from remote antiquity, in different parts of Aryan lands, side by side with different calendars pertaining to Xšaθra, having their years lunar, luni-solar or solar.⁸ Persians, in Achaemenian times, used a lunisolar calendar with the 19-year cycle; it can be inferred from Darius’ Bēstūn inscription. In the Sasanian times, two year-forms were treated by Persian astronomers, and each one had its own use in Persia (Ērānšahr), that is, the tropical year and the sidereal year.⁹

As for the first year form, the Dēnkird describes it as the time during which the annual seasons complete their course: the original season of the year is spring, when the sun reaches the first asterism of Aries (the first point of Aries), which is Perispar –the vernal equinox.¹⁰ The first paragraph quoted by Bērōnī, from the books of the Persian Magi, is this:

“The Persians who [from the remotest time] intercalated their years, marked out the four seasons of the year by their months, because of their nearness: Fravardīn-māh was the first [month]

⁶ cum primo die eius mensis, quem vocant Aegyptii Θουβοί, caniculae sidus exoritur. nam eorum annus civilis solum habet dies CCCLXV sine ullo intercalari. Censorini *De Die Natali Liber*, F. Hulsch, Lipsiae, 1867, 38.

⁷ . « L’année vague fut ... une institution essentiellement sacerdotale, appropriée aux cérémonies du culte ; et elle se maintint, consacrée par une pensée religieuse qui voulait que chaque fête passât successivement à tous les jours de l’année, et que chaque jour fût sanctifié. On ne saurait douter que les Égyptiens et les Perses n’aient bientôt connu la quantité dont elle devance l’année tropique, et appris, à un moment donné, à la raccorder avec le véritable état du ciel. » Édouard Dulaurier, *Recherches sur la chronologie arménienne*, Technique et historique, Paris, 1859, 3.

⁸ . The old Indians also used two kinds of calendars, one for sacrificial purposes (*kārman*- nt. ‘act ; rite as sacrifice, etc.’), and the other for civil calculation (*saṃ-khyā*- f. ‘reckoning, numeration, calculation ; number’). Cf., for example, the statement of Garga quoted by Somākara in his commentary of Jyotiṣavedāṅga of the Yajurveda and of the Ṛgveda, 5 : तेषां च सर्वेषां नक्षत्राणां कर्मसु कृत्तिकाः प्रथम मा चक्षते श्रविष्ठा तु संख्यायाः See also Bál G. Tilak, *The Orion or researches into the Antiquity of the Vedas*, Bombay, 1893, 30-31 ; G. Thibaut, « On some recent attempts to determine the antiquity of Vedic civilization », *Indian antiquary*, xxiv 1895, 98.

⁹ . As for the lunar calendar, see my “A Persian « Lunar » Calendar according to the Annals of Ĥamza”, *Er*, 2010.

¹⁰ . Dk iii, M 403 ud sāl ī zaman-vihēzagīg ī az sālān ī vas hamēnīd ōz abar kār ī az rōz abar hangāmān ī sāl cahār handāxtag. sāl hangām bun vahār, hān ī ka x^varšēd ō fradum x^vardag ī varrag, ī ast perispar, rased.

of summer, Tīr-māh the first [month] of autumn¹¹, Mihr-māh the first [month] of winter, and Day-māh the first [month] of spring.”¹²

In the *Book of the Crown*, written by a Persian scribe of the 8th century, some customs of the Persians during the two seasonal festivals of Mihragān in the month of Mihr and Nōgrōz in the month of Fravardīn have been described –seemingly according to the book *Ēvēn-nāmāg*. Bērōnī’s account is confirmed by this description:

“It is a duty towards the king to offer him presents on the occasion of Mihragān and Nōgrōz, because these two festivals mark the two seasons of the year. Mihragān occurs at the beginning of winter and the cooling season, while Nōgrōz marks the introduction of the heating season, however the latter is accompanied by circumstances which are not found in the first: especially the welcome of the new year, the commencement of the tax-collection, the nomination of [government] agents, the transfer [of the staff], the minting of silver and gold coins, the cleaning of fire-temples, the pouring out of water, the consecration of offerings, the construction of buildings, etc.”¹³

As the Persians reckoned their year as 365 days, its beginning continually receded by a quarter-day every year from spring equinox; then they added the quarters of a day in every 120 years as one complete month, when it has summed up thereto – the Romans added one day every fourth year (“leap” year). Then Nōgrōz remained around the summer solstice with the accuracy of more or less one month.

¹¹ . In Persian Tīr-māh also means the season of autumn. In the *Šāhnāma*, when describing the throne Tāgdēs, Firdōsī said thus (Š, M 7.154-158) :

چو اندر بره خور نهادی چراغ،	پسش دشت بودی و در پیش باغ.
چو خورشید در شیر جستی درشت،	مر آن تخت را سوی او بود پشت.
چو هنگامه تیر ماه آمدی،	گه میوه و جشن گاه آمدی،
سوی میوه و باغ بودیش روی	بدان تا بیاید ز هر میوه روی.
زمستان که بودی گه باد و نم،	بر آن تخت بر کس نه بودی دژم.

¹² . *Vestiges*, IX.2 :

« ان الفرس حين كانوا يكبسون سنهم، يعلمون الفصول الاربعة بشهورهم، لتقارب الأمر بينهما؛ فكان فروردین ماه

اول الصيف، وتير ماه اول الخريف، ومهر ماه اول الشتاء، و دى ماه اول الربيع. » الآثار، ٢٦٤

¹³ . « ومن حق الملك هدايا المهرجان والنيروز. والعلة في ذلك أنهما فصلا السنة. فالمهرجان دخول الشتاء وفصل

البرد، والنيروز إذن بدخول فصل الحر. إلا أن في النيروز أحوالاً ليست في المهرجان؛ فمنها استقبال السنة، وافتتاح الخراج، وتولية العمال، والاستبدال، وضرب الدراهم والدنانير، وتذكية (تذكية) بيوت النيران، وصب الماء، وتقريب القربان، وإشادة البنيان، وما أشبه ذلك. » كتاب التاج في أخلاق الملوك للمؤلف للجاحظ، ١. زكى باشا، القاهرة، ١٩١٤، ١٢٤-١٢٨ / ف.

عطوي، بيروت، ١٩٧٠، ١٤٨.

1 y = 356.25^d or 365; 15^d

But this year-length is a rough approximation to the mean “tropical” solar year: “These [receded hours] become *roughly* (*društagīhā*) 1 day in 4 years, 10 days in 40 years, 1 month in 120 years”.¹⁴ Indeed the value adopted for the tropical year by Persian astronomers is a little less than that.¹⁵ For this reason, the intercalation period for this kind of solar year may be 124 years and occasionally 125 years.

The religious year form is the “ideal” year but does not correspond with the natural seasons of a year, and its months do not have a fixed position in the yearly cycle; while the calendar system with a periodic cycle of intercalations is highly convenient for agriculture and other seasonal works as well as for some economic and administrative purposes. Indeed, the intercalary month is added to bind this calendar which gives a convenient means of coordinating human activities and the natural passage of days and seasons to the solar tropical year. The neglect of seasons implies the displacement of Nōgrōz, that is, the New Day of the year and also the starting date for the assessment of the annual tax-bill and collection of the first tranche –this happened during the period of Arab invasion and forced the Persian landed gentry (*dahegān*) into bankruptcy. About the consequences of the neglect of seasons the Dēnkird gives a clear exposition. It reads as follows:

“As the bond of these four seasons of the year is [linked] to the motion of the sun through the zodiacal signs, by the collection of hours at the end of each year [of 365 days] much delayed time in [the lapse of] years would reach (i.e., accumulate) to days, and days to months, and months to years, and years to greater divisions of time; and thereby the seasons may have to be relinquished. With the relinquishment of seasons [will be relinquished] what is connected naturally with the four seasons, such as the generation, growth, increase, ripening, and maturing of grain[-plants] and [fruit-]trees, the movement of people and the journey of kings to the summer and winter

¹⁴ . Dk iii, M 402 ud društagīhā ped cahār sāl ēk rōz, ud ped cahal sāl dah rōz, ud ped sad-vīst sāl ēk mäh.

¹⁵ . Cf. Bd 161 x^varšēd az hān x^vardag ī varrag ī ped bunīh bē raved ped sisad-šast-panz rōz ud panz zamān ud x^vardag ī [zamān] ast ī sāl-ē abāz ō hān gyāg rased.

quarters¹⁶, the change of winds and the steering in the seas which is appointed according to the winds.”¹⁷

In this way, the year form with a regular cycle of intercalations which, because of keeping pace with the seasons, is in compliance with the tropical year may be called “civil” (Pers. *šahrīg*) or even “royal” (Pers. *šāhīg*). However, it has been considered, by most of the modern authors on the subject, as the religious year (called the *vihēzagīg* year), and its first month, Fravardīn māh, has been taken as the first month of spring.¹⁸ If Nōgrōz were in early spring, then how the land-tax could be collected before harvest time? Moreover, Pers. *vihēzagīg* means ‘moving; movable’, and not ‘intercalary’ (as supposed by some authors); Pers. *vihēz(ag)* ‘moving, displacement; departure’ comes from *vihēz-*: *vihist* ‘to move, pass from one place to another’ (Parth. *wihīšt*, Kurd. *guhēz-*).¹⁹ Zādspram uses correctly the word *vihēzag* in the context of the recession of one day every 4 years of 365-days:

*harv cahār sāl rōz-ē vihēzag, nē abzōn; ped šaš-hazār sāl cahār sālān x̄māndag*²⁰ *baved.*

‘In every four years there is one day which is displaced and is not added; in six thousand years there will be four receded years.’²¹

¹⁶ . According to Wèi shōu (506-572 A.D.): “The [Persian] king has in his realm a dozen or more small (royal) residences just like the (travel) imperial palaces in China. Every year in the fourth month [of the lunar year] he leaves [the capital] and goes on a tour, and in the tenth month he comes back [to the capital].” See my “The Wèi-shū and the Persian calendar”, *Er*, 2012.

¹⁷ . Dk iii, M 404 cōn ēn cahār hangām ī sāl bann ō ravišn ī x^varšēd ped axtarān, andar sālān vas drang az hamīh ī zaman ī abar ēk sāl sar ō rōzān, rōzān ō māhān, māhān ō sālān, sālān ō brīnagān ī meh rasišnīg, ud pediš hangām hilišnīg. abāg hilišn ī hangām hān ī ō cahār hangāmān cihriḡ peyvastag, cōn bavišn ud rōyišn ud vaxšišn ud pezāmišn ud rasišn ī jōrdāyān ud urvarān, vihēz ī mardōm ud ēvarz ī x^vadāyān ō ōstānīgān ī hāminīg ud zimizīg, vardišn ī vādān ud ravišn ī ped zrehān ī ō vādān handāxtag. See the earlier study of the 419th chapter of the Dēnkird iii by Sheriarji D. Bharucha: “Pāzend and English versions of a chapter of the Pahlavi Dīnkard, related to the solar and luni-solar years in the Zoroastrian religion”, *The K. R. Cama Memorial Volume*, Bombay, 1270/ 1900, 12-28.

¹⁸ . About the *vihēzagīg* month and Fravardīn as the first month of spring in the 25th chapter of the Bundahišn, I have discussed elsewhere: abar sāl ī dēnīg, *Er*, 2011.

¹⁹ . Cf. MSR 9 vihēz ī az gyāg ō gyāg; Dk vii M 644 vihēz ī yaštfravahr zardušt; MX 49.27 vihēzag ī x^varšēd ud māh; ŠGV 5.44 jumbišn vihēzag ī tuhīg

²⁰ . The reading *cahār sāl humānāg* does not make sense. Cf. Arab. الموانيد ‘arrears ; back tax’ (from Pers. *māndag*).

²¹ . VZ 34.49.

Indeed, both “solar” year forms, one without intercalation and the other with one intercalary month every (around) 120 years, are *vihēzagīg*, as it is said in the Dēnkird:

xʷaršēdīg sāl do ēvēnag: rōz vihēzagīg ī andar sāl, ud zaman vihēzagīg ī az sālān.

‘The solar year is of two kinds: The [year in which] the days move in the year; and [the year in which] the hours move from year to year.’²²

Bērōnī in his *Canon* states that the Persian Magi call the “intercalary year”²³ بهرك. This word can be read in two ways: *bihēzag* (as in the edited book) and *nihanzag*. As we said, the word *b/vihēzag* in no way means ‘intercalation’ or ‘intercalated’. But what about the word *nihanzag*? *nihanzag* comes from *nihanz-/ nihenz-* ‘to hold back, retain; retreat, recede’ (Parth. *nihanj-/ nihenj-*), and means ‘retreat, holding back’. Then this is a convenient word for intercalation, because intercalation, through inserting a day or a month, holds back the year to a certain position. Fortunately the same word, in its Parthian form, exists in Armenian: նահանջ /*nahanj*/ ‘retreat, rout’, adj. ‘bissextile’; նահանջ տարի /*nahanj tari*/ ‘leap-year’ (նահանջաւ) ; նահանջաւկաւ /*nahanjakan*/ ‘intercalary’; նահանջում /*nahanjumn*/ ‘intercalation; solstice’. There is no doubt that بهرك in Bērōnī’s book should be read نهنك /*nihanzag*/ meaning, in this context, ‘intercalation’.

§ 1.3. The Persian astronomers were familiar with another year form, the “sidereal”, connected with the annual risings and settings of stars, and used it in their ephemerides, goal-year texts, etc., and (probably) took the point opposite Spica as the zero point (sidereal longitude). The intercalation scheme of 116-year period is related to this “sidereal” year. In one passage of his *Chronology*, Bērōnī alludes to it.²⁵ A similar statement is found in the *Prolegomena to the Study of Arab Historians*, written by Ṣafḍī. It reads as follows :

²² . Dk iii, M 402.

²³ . Arabic سنة كبيسة ≈ Syr. ܥܘܠܐܡܐ ܚܘܨܘܠܐ ‘intercalary year, leap-year’; Arabic كبيسة ≈ Syriac ܚܘܨܘܠܐ ‘intercalated’.

²⁴ . *Canon Masudicus*, I-III, Hyderabad, 1954, I, 90: « و أما الطبقة الثانية فهم الفرس في الموسية و قد سموا سنة الكبيسة بميزك ... » البيروني، القانون، عبدالكريم جندی،

بيروت، ١٤٢٢، ١: ١٣٨

²⁵ . *Vestiges*, II.5 :

« ... فألحقوا الشهر التام بها، في كل مائة و ست عشرة سنة. » الآثار، ١٥

“The Persians intercalated the excess of one month arising on the difference between their year and the solar year, every 116 years. This embolism, despite its length, is more exact than that of the Romans, because it approaches more the result given by the calculation of the excess of the solar year. Then, when Islam came, they stopped to make use of embolism and it was not observed anymore and this was prejudicial to the [Persian] people.”²⁶

Some other authors also give the intercalation period as 116 years.²⁷ Then the length of the interval is:

$$116 y = (116 \times 365 + 30)^d = 42370^d$$

$$1 y = 365.25862^d, 365^d 6^h 12^m 25^s \text{ or } 365; 15, 31, 2^d$$

²⁶ . « Prolégomènes à l'étude des historiens arabes par Khalîl ibn Aibak Aṣ-Ṣafadî », M. Émile Amar, *Journal Asiatique*, Mars-Avril 1911, 278 :

« وكانت الفرس تكبس الفضل الذي بين سنتها وبين سنة الشمس في كل مائة وستة عشر سنة شهراً وهذا الكبس على طوله اصح من كبس الروم لانه أقرب الى ما يحصله الحساب من الفضل في سنة الشمس. فلما جاء الاسلام، عطل ذلك ولم يعمل به فأضّر بالناس ذلك.»

²⁷ . For example, Abū Naṣr of Qum, a Persian astronomer, relates in his *History of the Caliphs* (preserved in the *History of Qum*) a conversation between Al-Mu'tamid and his Vizier 'Ubayd Allāh ibn Sulaymān concerning the commencement of the annual land tax for the purpose of easing the tax burden of people who suffered at that time; the vizier referred to the rule of an intercalary cycle of an extra month every 116 years in Persian times:

« ابو نصر حسن بن علی قمی منجم ایضا در کتابی که او را بوده مترجم و مشهور به کتاب تاریخ خلفا، و روزگاریها و عیدهای ایشان یاد کرده، حکایت نموده که: عجم را افتتاح و ابتدای خراج ایشان در هر سالی در ماه فروردین بوده در اول سال ایشان. زیرا که آفتاب در این هنگام (ای، هنگام پارسین) به برج حمل در می آید در اول آذر، و غلات و خضریات در نوروز مستدرک و رسیده می شود. پس از این جهت افتتاح خراج در این وقت می کردند. پس چون کبیسه عجم در اسلام بیفتاد، فصول سال ایشان به گردیدند، و ادراک غلات واپس افتاد. پس از این جهت بدیشان زحمت و رنج رسید، و در ادای خراج به تنگ آمدند سبب واپس افتادن ادراک غلات و ارتفاعات از ابتدای دادن خراج، و این معنی بدین دستور جاری بود تا آخر روزگار معتمد. چون مردم به تنگ آمدند، شکایت به نزدیک عبیدالله بن سلیمان وزیر بردند. عبیدالله ایشان را وعده داد و قبول کرد که در این کار به جهت ایشان نظر فرماید و اندیشه کند. تا روزی از روزها معتمد در بعضی از منزهات و بستانها و عشرتخانهها به عیش و نشاط و طرب مشغول بود. پس وزیر حدیث در پیوست و عنان سخن بدین کشید که: “چگونه باشد حال طایفه و قومی که در این وقت و هنگام از ایشان طلب خراج کنند، و ابتدای آن از این وقت گیرند، و حال آن که غلات در صحاری به نسبت همچو گیاه اند و میوها برگها اند؟” معتمد چون این سخن از وزیر خود عبید به شنود، فرمود که: “در ایام قدیم در این وقت افتتاح و ابتدای گزاردن خراج نه بوده؟” عبیدالله گفت: بلی، فاما ادراک غلات در آن زمان در این وقت (نوروز) بوده است، زیرا که سال طبیعی که در آن فصول چهارگانه تمام می شوند، سیصدوشصت و پنج روز و ربع روزی است و کسری، و حال آنکه فرس بدین ربع روزها در سالهای خویش کبیسه می کردند، پس به هر صدوشانزده سال ماهی زیاده می کردند. پس بنا بر این همیشه در وقت نوروز ایشان ادراک غلات بوده. پس چون پادشاهی عجم زوال پذیرفت، و کبیسه ربع ایشان بیفتاد، ادراک غلات واپس افتاد به هر چهار سال یک روز.» تاریخ قم، حسن بن محمد قمی، (ج. تهرانی) تهران، ۱۳۱۳، ۱۴۵-۱۴۶.

Of course, this year length is the closest possible approximation to that of the Persian sidereal year according to the *Zīg ī Šahriyārān*. Bērōnī, in dealing with the Sogdian festivals, states: “The ancient Persians used a solar year of 365 days and [a bit] more than a day shift, and it was their universal practice to reckon these 6 hours *plus* the fraction of an hour, and disregarded the minutes in reckoning.”²⁸ *is* *جزء من ستين جزءاً من ساعة*²⁸ rendered in the English translation as “the 1 minute as a unit”.²⁹ However, Bērōnī uses *daqīqa* for ‘minute’. The expression used by him renders exactly Pārsīg *xʷardag ī zamān* ‘the fraction of an hour’.³⁰ In another passage, he gives the length of 365^d 6^h 12^m for the solar year according to the Persians.³¹

$$1 y = 365 ; 15, 30^d$$

This corresponds to the length of one sidereal year in the Pauliśa-Siddhānta.³² The year length in the Royal astronomical tables (*Zīg ī Šāh*) is a little longer³³:

$$1 y = 365.259^d \text{ or } 365^d 6^h 13^m = 365; 15, 32, 30^d$$

This number is almost the same as the year length of the Persians mentioned according to the Abū Maʿšar. Hāšimī says:

“If we want the determination for solar years we divide the world-cycles by the solar cycles. There will come out for us the fractional days of a year. If we do that with the Persian cycles

²⁸ . *Vestiges*, X.2.

« وذلك انّ الفرس الأول كانوا يعملون على انّ سنة الشمس ثلاثمائة وخمسة وستون يوماً، وأكثر من ربع يوم بجزء من

ستين جزءاً من ساعة، و يتعاهدون جبر تلك الزيادة، على ربع يوم اليه. » الآثار، ٢٩١

²⁹ . *The Chronology of Ancient Nations*, E. Sachau, London, 1879, 220.

³⁰ . Cf. Dk iii, M 402 ud zaman vihēzagīg ī az sālān hān ī az šaš zaman ud xʷardag ī zaman ī abar sar ī 365 rōz hamih harv sāl ‘The hours moving from year to year consist of the 6 hours and the fraction of an hour which accumulate at the end of 365 days.’

³¹ . *Vestiges*, II.5 :

« ... ومن خمس الساعة الذي يتبع ربع اليوم ... » الآثار، ١٥

³² . See Varāhamihira, PS 3.1. Cf. also the Indian sidereal year according to Āryabhaṭa : 365.258605^d.

³³ . Bērōnī states : “93 ¼ degrees is the surplus of the solar cycle over the whole according to the Persians.” *Vestiges*, VI.60 (*Chronology*, 121):

« ثلاثة وتسعين جزءاً وربع جزء، و هو زيادة الدّور الشمسيّ على الأيام الصّحاح عند الفرس. » الآثار، ١٣٥

Indeed, 93 ; 15° corresponds to 6 ; 13^h, and to the year length of 6, 5 ; 15, 32, 30^d. See also the *Introduction to Astrology* of Abū Naṣr:

« و اگر طالع اصل به زیج خوارزمی کرده باشند سالهای کامل مولود را در نودوسه درجه و دو دقیقه

ضرب کن؛ و اگر اصل به زیج شاه باشد آن را در نودوسه درجه و پانزده دقیقه، و بیوگن از وی دوری

همیشه، آنچه حاصل آید [کمتر از دور] درجه طالع اصل از وی بیوگن به مطالع آن شهر کی مولود بده

باشد چنانک به گفته آمدست. » ابونصر حسن بن علی قمی، ترجمه المدخل الی علم احکام النجوم، ج. اخوان

زنجان، تهران، ۱۳۷۵، ۲۳۸

and their days there will come out for us for the days of a solar year, 365 days and 15 minutes and 32 seconds and 24 thirds.”³⁴

$$1 \text{ y} = 365.259^d \text{ or } 365; 15, 32, 24^d$$

Notice that the value of the Persian Sidereal year is close to the length of an anomalistic year. Anyway, the Persian astronomers were familiar with the difference between the tropical and “anomalistic”³⁵ year length, rather used sidereal longitudes, and prescribed the intercalation period of 116 years.

§ 2. Let us now turn to the official supervising of the last intercalation, and examine the date of the event. Bērōnī refers to it on a number of occasions.

§ 2.1. He first reports that:

“The Persians had already begun to neglect their intercalation nearly seventy years before the perdition of Yazdegird. Because at the time of Yazdegird son of Šābūr they had intercalated into their year two months, one of them as the necessary compensation for that space of time, by which the year had moved backward (it being too short). The five Epagomenæ they put as a mark at the end of this intercalary month, and the return had just come to Ābān Māh, as we shall explain hereafter. The second month they intercalated with regard to the future, that no other intercalation might be needed for a long period. Now, if you subtract from the sum of the years between Yazdegird son of Šābūr and Yazdegird son of Šahriyār 120 years, you get a remainder of nearly –but not exactly – 70 years; there is much uncertainty and confusion in the Persian chronology. The *portio intercalanda* of these 70 years would amount to nearly 17 days.”³⁶

³⁴ . ‘Alī ibn Sulaymān al-Hāšimī, *The Book of the Reasons behind Astronomical Tables*, F. I. Ḥaddād & E. S. Kennedy, New York, 1981, 129.

« و اذا اردنا معرفة سنة شمسية قسمنا ادوار العالم على ادوار الشمس فيخرج لنا ايام كسور سنة فاذا فعلنا ذلك بادوار

الفرس و ايامهم خرج لنا ايام سنة شمسية ٣٦٥ يوما و به دقيقة و لث ثالثة و كد ثالثة.» كتاب في علل الزيجات، ١٠٩

³⁵ . Indeed « sidereal » year, as Pillai says about the Indian solar year: « Strictly speaking, a sidereal year, but the designation anomalistic year is more suitable at this stage in order to lead up to the sun’s anomaly. » S. Pillai, *Indian Chronology (Solar, Lunar and Planetary)*, Madras, 1911, 2.

³⁶ . *Vestiges*, III.46 (*Chronology*, 38) :

« وذلك أنّ إهمال الفرس كبيستهم، كان قبل هلاك يزدرج بن شهريار بقريب من سبعين سنة، لأنهم كانوا كبسوا السنة

في زمان يزدرج بن ساپور بشهرين، أحدهما لما لزم السنة من التأخر، وهو الواجب. ووضعوا اللواحق خلفه علامة له،

وكانت التوبة لأبان ماه، كما سنذكر، والشهر الآخر للمستأنف، ليكون مفروغاً منه الى مدّة طويلة، فإذا أسقط عن السنين

Another passage in Bērōnī's *Chronology* gives more detailed information on this last intercalation:

“Being afraid that there might arise uncertainty as to the place where the intercalary month would have again to be inserted, they transferred the five Epagomenæ and put them at the end of that month, to which the turn of intercalation had proceeded on the last occasion of intercalating. And as this subject was of great importance and of general use to high and low, to the king and to the subjects, and as it is required to be treated with knowledge, and to be carried out in conformity with nature (i.e. with real time), they used to postpone intercalation, when its time happened to occur at a period when the condition of the empire was disturbed by calamities; then they neglected intercalation so long, until the day-quarters summed up to two months. Or, on the other hand, they anticipated intercalating the year at once by two months, when they expected that at the time of the next coming intercalation circumstances would distract their attention therefrom, as it has been done in the time of Yazdegird son of Šābuhr, for no other motive but that of precaution. That was the last intercalation which they carried out, under the superintendence of a *dastōr* (‘religious authority; having mandate’), called Yazdegird ī Hazārīg. Hazār was an estate in the district of Staxr in Persis, from which he received his name. In that intercalation the turn had come to Ābān Māh; therefore, the Epagomenæ (اندرگاه) were added at its end, and there they have remained ever since on account of their neglecting intercalation.”³⁷

These passages inform us about the method of intercalation and also the time when it was carried out:

1. The Persians intercalated two months, one for making the calendar follow the “solar” year, and the other for

التي بين يزديجرد بن سابور و بين يزديجرد بن شهريار مائة وعشرون سنة، بقي بالتقريب سبعون سنة لا بالتحقيق. فإنّ تواريخ الفرس مضطربةً جداً، ويكون حصّة هذه السبعين سنة من الأرباع، قريباً من سبعة عشر يوماً.» الآثار، ٤٠

³⁷ . *Vestiges*, V.9 (*Chronology*, 55-6) :

« و خافوا اشتباه الأمر عليهم في موضع التوبة. فأخذوا ينقلون الخمسة الأيام، و يضعونها عند آخر الشهر الذي انتهت إليه نوبة الكبيسة؛ و لجلالة هذا الأمر و عموم المنفعة فيه للخاصّ و العامّ و الرعيّة و الملك، و ما فيه من الأخذ بالحكمة و العمل بموجب الطّبيعة، كانوا يؤخّرون الكيس إذا جاء وقته؛ و أمر المملّكة غير مستقيم لحوادث، و يهملونه حتّى يجتمع منه شهران، أو يتقدّمون بكيسها بشهرين، إذا كانوا يتوقّعون وقت الكيس المستأنف ما يشغل عنه؛ كما عمل في زمن يزديجرد بن شابور أخذاً بالاحتياط، و هو آخر الكبائس المعمولة، تولاه رجل من الدّستورين يقال له يزديجرد الهزاريّ - و هزاز ضبعة من كورة إصطخر بفارس ينسب إليها -، و كانت التوبة في تلك الكبيسة لأبان ماه، فألحق الاندركاه بأخره، و بقيت فيه لإهمالهم الأمر.» الآثار، ٥٤

anticipating the next intercalation; and they transferred the epagomenae from the month of Mihr to the month of Ābān.

2. Bērōnī points out two other points: One, the intercalation was done in the time of Yazdegird I; and the other, the next “neglected” intercalation should fall 50 years after the death of Yazdegird III (he ruled for twenty years over Ērānšahr).
3. He gives the name of the person having mandate and architect of carrying out this “intercalation”, a certain Yazdegird of Hazār.³⁸

The Persian source of Bērōnī’s account should have related that Yazdegird carried out the intercalation without mentioning the name of the king under whom it had taken place. The homonymy of the astronomer Yazdegird and a Persian king of the fifth century could lead Bērōnī to confusion ; and he, without bothering to check for the accuracy of his account, attributed the last intercalation to the Persian king Yazdegird son of Šābūr.

Now we verify the date of this event. Bērōnī gives the following rule which also determines the date of the last intercalation:

“If we want to know the intercalation, as practiced by the Persians before the decline of their empire, we take the Persian years from the end of the reign of Yazdegird, which event is the epoch of the *Æra Magorum*, and add thereto 70 ... The sum we divide by 120. The quotient is the number of intercalation that ought to have been carried out since the time when they commenced to neglect intercalation. Now we take for the total sum of the years of the era a number of months corresponding to the number of intercalations. If, then, these months make up complete years, without giving a remainder, the year is a leap year approximately, for there is confusion in their chronology. But if there is a remainder of month, the year is a common year. Thereupon we add the leap-months we have got to the beginning of the year in question, and we find Nōrōz on that day to which this calculation brings us. So Nōrōz comes again to be there, where it used to be in the time of the Xusrō [kings], when it used

³⁸ . It seems that Yāqūt have drawn his information from Bērōnī’s book, the *Vestiges* (see Sachau, *Chronology*, 383) :

«الھزار: قرية بفارس من كورة إصطخر، ينسب إليها يزديجرد الھزاري آخر من عمل كبس السنين في أيام الفرس في أيام

يزديجرد بن سابور.» ياقوت الحموي، معجم البلدان، ٥، بيروت، ١٣٩٧/١٩٧٧، ٤٠٤

to coincide with the summer solstice as calculated by their astronomical tables.”³⁹

The day 1 of the Pārsīg era (or, *māh ī fravardīn rōz ī ohrmazd ī vīst sāl pas az yazdegird*), corresponds to A. D. 652, June 11, Monday (JD 1959362.5/ KD 1370897.5).⁴⁰

Let Y_p be a certain Pārsīg year, then n will be the number of neglected intercalations:

$$n = [Y_p + 70]_{120}$$

We see that the first intercalation after king Yazdegird son of Šahriyār should be carried out in the year P. E. 50 or A. D. 702. Then, 240 years before it, A. D. 462, will be the date of the last intercalation. This date confirms that of the first passage cited from the *Chronology*. However, this date indicates that the double intercalation must have been taken place during the first years of the Persian king Pērōz, and not at the time of king Yazdegird son of Vahrām.

§ 2.2. In his later book, the *Canon*, Bērōnī reviews the Persian intercalation scheme, and as for the last intercalation he states:

“Then they (the Persians) state that: The last intercalation was in the time of Pērōz son of Yazdegird who was one of their kings, who intercalated two months: One was necessary for the past, and the other was taken by way of precaution on account of

³⁹ . *Vestiges*, VII.86 (*Chronology*, 184-85) :

« وإن اردنا معرفة الكبيسة، التي كان الفرس يستعملونها قبل زوال ملكهم، أخذنا سني الفرس من زوال ملك يزدجرد، و هو تاريخ الجوس، و زدنا عليها سبعين سنة، للعلّة المذكورة في أوائل الكتاب، و قسمنا ما اجتمع علي مائة و عشرين، فما خرج، فهو عدد شهور الكبائس، من لدن وقت الإهمال؛ فنمیز من جملة التاريخ، شهورا علي عدد الكبائس؛ و ننظر، فإن نفذ السنون، و لم يبق منها شيء، فالسنة كبيسة بالتقريب، لاضطراب التواريخ؛ و إن بقي شيء، لم تكن كبيسة؛ ثم نزيد ما خرج من شهور الكبائس، على أول سنتنا، و نجعل النيروز حيث ما ينتهي بنا؛ فيكون موقعه، بحيث كان يقع في زمان الأكاسرة، و قد كان يتفق حينئذ، مع الانقلاب الصيفي المحسوب بزيجاتهم. » الآثار، ٢٤٠-٢٤١.

⁴⁰ . He says : The era of the Perso-Aryans (‘era of the Magi’ ‘Æra Magorum’) is from the year of the death of Yazdegird apart from the years of his reign the length of which was 20 years. See the *Canon* :

« و أما تاريخ الجوس فإنه من سنة مهلك يزدجرد دون سنة ملكه و كانت مدته عشرين سنة فإذا نقصت من تاريخ قيامه بقي تاريخ تلفه و كان مقتله بمرو علي اقتراب من السغد، فاستعمل مجوسها وقته و لكن مجوس ما وراء النهر مخالفون لجوس خراسان و فارس في الاعتقاد بحيث يكاد يسبق إلى الوهم أن داعيهم غير داعي أولئك، و سنوهم مبتدئة من النيروز الكبير المتأخر عن نوروذ الملوك خمسة أيام و لذلك يخالف شهورهم شهر الفرس إلى أول آذارماه ثم يتفق إلى أول اسفندارمذماه، و الخمسة الأيام الزائدة ملحقة بالشهر الثاني عشر من شهورهم معدودة من جلته فلذلك نقصنا من تاريخ يزدجرد لأجلهم عشرين سنة و خمسة أيام. » القانون، ١ : ١٧٤

the opinion of the king about the decline of the [Aryan] kingdom, that was in the process of decay.”⁴¹

Again, he states :

“... Between Pērōz, forefather of Xusrō Anōšervān, who undertook the last intercalation, and Yazdegird, there are close to 170 years, and if it be added to them, from the rest of years until the intercalation, 90 years, then there will be 260 years.”⁴²

As we see, Bērōnī has made two adjustments to his previous account:

1. Indeed, the double “intercalation” has been done in the time of Pērōz son of Yazdegird.
2. He yields the date nearly A. D. 482. Then the rule for determining the number of neglected intercalations after the death of Yazdegird III is:

$$n = [Y_p + 50]_{120}$$

§ 3. However, a few questions concerning this last intercalation remain to be resolved:

1. Why does the difference between two kinds of the “solar” year, the religious and the civil, after this event cease to exist?

Bērōnī does not say anything about the relation of “religious” and “civil” calendars after this intercalation.

2. Why is there a difference of five days between the first day of the Persian year and that of the Sogdian year?

The reason of the five-day difference between the Persian calendar and the Sogdian calendar remains obscure. The later Sogdians did not know when exactly the Sogdian calendar disrupted its agreement with the Persian calendar.

⁴¹ . *Canon*, I, 132 :

« ثم ذكروا أن آخر الكبائس كانت في أيام فيروز بن يزدجرد من ملوكهم، و أنه كبس شهرين أحدهما استحقاق بالماضي و الآخر استئناف للمستأنف أخذا بالاحتياط لما رأى الملك إلى الزوال و الذين بصدد الانحلال. » القانون، ١ : ١٦٧

⁴² . *Canon* , I, 91 :

« والوجه الآخر أنه ذكر في أخبارهم عن زرادشت أنه كان بقي في أيامه إلى تمام الكبيسة مقدار من السنين لم يتحققه و لا شك في أنه أقل من دورها و بين فيروز جد أنوشروان الذي تولى الكبس الآخر و بين يزدجرد قريب من مائة و سبعين سنة، فإذا انضاف إليها من تلك البقية إلى الكبيسة تسعون سنة كانت السنين المائتين و الستين. » القانون، ١ : ١٣٩

The answer to these questions will be found in the last reform of the Persian calendar and the double intercalation scheme during the reign of Pērōz.

§ 3.1. Let us see the method of Yazdegird the astronomer for carrying out the double intercalation⁴³:

The first month of the civil calendar, Fravardīn, corresponded to the month of Vahman of the religious year on the threshold of the intercalary year.

<i>šahrīg</i> (civil)	<i>dēnīg</i> (religious)
Fravardīn	Vahman
Urdevahišt	Spendārmed
	Panzag
Hurdād	Fravardīn
Tīr	Urdevahišt
Amurdād	Hurdād
Šahrever	Tīr
Mihr	Amurdād
Panzag	
Ābān	Šahrever
Ādur	Mihr
Day	Ābān
Vahman	Ādur
Spendārmed	Day

If we take the year before the double intercalation A. D. 461, then the first day of the civil year will be May 30, Tuesday, and that of the religious year (Nōgrōz) August 3, Thursday. Notice that the summer solstice is 21 June 461.⁴⁴

If two months be added between the months Mihr and Ābān of the civil calendar, the months of the civil and religious calendars will coincide with each other with only a five-day difference.

<i>šahrīg</i>	<i>dēnīg</i>
Fravardīn	Vahman
Urdevahišt	Spendārmed
	Panzag

⁴³ . It could be valid for the years 461-63 as well as 481-483.

⁴⁴ . But if we take the year before the double intercalation A. D. 481, then the first day of the civil year will be May 25, Monday, and that of the religious year (Nōgrōz) July 29, Wednesday. Notice that the summer solstice is 20 June 481.

Hurdād	Fravardīn
Tīr	Urdevahišt
Amurdād	Hurdād
Šahrever	Tīr
Mihr	Amurdād
Anāmag ⁴⁵	Šahrever
Anāmag	Mihr
Panzag	
Ābān	Ābān
Ādur	Ādur
Day	Day
Vahman	Vahman
Spendārmed	Spendārmed
	Panzag

In this “intercalary” year, two “religious” months of Spendārmed occur, and as the place of the “religious” Epagomenæ is after Spendārmed, hence the five-day difference of the beginnings of the “religious” and “civil” years. By omitting the last set of epagomenal days and transferring the Epagomenæ to the month of Ābān Yazdegird the astronomer made the religious calendar to coincide with the civil one. The X^vārazmians and Sogdians did not follow this “heresy” –nor the Armenians. For this reason, the Persian year commenced five days before the X^vārazmian/ Sogdian year, and the first day of the month Nāwsārcī (Nausarδīc) fell on the sixth day (Hurdād) of the month Fravardīn, and the beginning of the calendar year in Transoxiana and Sogdiana and Armenia fell 5 days after the Nōgrōz in this “reformed” Persian calendar.

The calendar table after the double intercalation will be thus:

Fravardīn
 Urdevahišt
 Hurdād
 Tīr
 Amurdād
 Šahrever
 Mihr
 Ābān
 Panzag
 Ādur
 Day

⁴⁵ . Pers. *anāmag* ‘without name, intercalary month’, cf. OPers. *anāmaka*, Skt. *anāmaka*.

Vahman
Spendārmed

For the year 463-64, we know that Nōgrōz was on Monday, 29 July 463, and the summer solstice on Friday, 21 June 463. As we see, one intercalary month has been anticipated.⁴⁶

Notice that, during the reign of Pērōz, some adjustments were made concerning the computation of the mean and true longitudes of the planets, and the “Yuga” system. These subjects which inspired the Indian astronomer Āryabhaṭa will be discussed elsewhere in another essay.

Raham Asha

Māh ī Tīr rōz ī Vahman sāl 1385 pas az Yazdegird (18 October 2015)

⁴⁶ . For the year 483-84, we know that Nōgrōz was on Sunday, 24 July 483, and the summer solstice on Tuesday, 21 June 483.